Universal properties for linelike melting of the vortex lattice

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Abstract

Using numerical results obtained within two models describing vortex matter (interacting elastic lines (Bose model) and uniformly frustrated XY-model) we establish universal properties of the melting transition within the linelike regime. These properties, which are captured correctly by both models, include the scaling of the melting temperature with anisotropy and magnetic field, the effective line tension of vortices in the liquid regime, the latent heat, the entropy jump per entanglement length, and relative jump of Josephson energy at the transition as compared to the latent heat. The universal properties can serve as experimental fingerprints of the linelike regime of melting. Comparison of the models allows us to establish boundaries of the linelike regime in temperature and magnetic field.

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It is now well established that the vortex lattice in a clean superconductor melts in a first-order transition due to thermal fluctuations. The melting transition has been observed in a number of experiments using different techniques, including transport[1–3] and direct measurements of the magnetization jump[4–6] and the latent heat[7,8] associated with the melting transition.

Melting of the vortex lattice in real superconductors is a complicated phenomenon. Several different regimes exist depending on the parameters of the superconductors and the magnetic field. The simplest and most natural picture of the melting transition in three-dimensional superconductors is that a low temperature crystal of straight vortex lines transforms into a liquid of entangled lines.[9] In real layered superconductors this picture does not work either at too high or too low fields. At high fields, the vortex lattice is more adequatly described as a system of weakly interacting two-dimensional lattices. Melting in this situation occurs in a pointlike way and is accompanied by evaporation of vortex lines.[10] At low fields, on the other hand, the melting temperature approaches the fluctuation region and vortex degrees of freedom start to interact with other superconducting degrees of freedom (spin wave excitations and vortex loops). The contribution of these degrees of freedom to the latent heat of the melting starts to increase abruptly as the temperature aproaches T_c possibly leading to a significant change in the nature of the melting transition.[11,22]

As a quantitative theory of vortex lattice melting has yet to be developed, a large amount of work has been invested into numerical simulations of the vortex system. A complete picture can be worked out by direct simulation of the Ginzburg-Landau functional, but this would require enormous computational effort. Rather a number of simplified models have been used, such as interacting elastic lines (or the Bose model)[12–17], the frustrated XY-model[18–23], the Lattice London Model (or lines and loops)[24–26], and the Lowest Landau Level approximation.[27,28] Being approximations, all these models have their limited regime of applicability where they can be expected to make reliable predictions for the melting transition and for the properties of different vortex phases. Unfortunately, it is difficult even to make reliable analytical predictions for the range of applicability of a specific model. In this work we therefore use a more pragmatic approach of comparing the numerical results for two rather different models: the Bose model and the XY-model. We find that the results obtained from these models agree over a wide range of fields and temperatures, where the melting transition can be described as *linelike*. We show that in the linelike regime the transition is characterized by a set of *universal properties*, which can serve as fingerprints of this regime. Comparison of the models also allows us to establish boundaries of the linelike regime.

I. DESCRIPTION AND COMPARISON OF THE MODELS

The phenomenology of superconductors is based on the Ginzburg-Landau model, which expresses the energy of superconductor via the magnetic induction, $\mathbf{B}$, the modulus of the order parameter, $|\Psi|$, and its phase, $\phi = \arg\Psi$. The model is completely defined by the London penetration depth λ, the coherence length ξ, and the anisotropy factor γ. Since the high-T_c materials are strongly type II superconductors, i.e., $\kappa = \lambda/\xi \gg 1$, the modulus of the order parameter is suppressed only in the vicinity of vortex cores. This allows us to describe superconductors within the *London approximation* which neglects

fluctuations in the modulus of the order parameter everywhere except in the cores of the vortices. The approximation breaks down in the vicinity of the upper critical field H_{c2} and in the fluctuation region near the transition temperature T_c. The London approximation is used by both models studied in this paper. The frustrated XY-model then makes two further approximations: It neglects the fluctuations in the magnetic induction, usually called the *frozen-field* or *infinite-lambda* approximation, and introduces a lattice on which the phase of the order parameter is defined. The model desribes layered superconductors only in terms of the phase distribution $\phi(\mathbf{n})$, defined on a three-dimensional grid $\mathbf{n} = (n_x, n_y, n_z)$. The energy functional of this model is given by (see, e.g., Ref. 29).

$$\mathcal{F}[\phi(\mathbf{n})] = \sum_{\mathbf{n}} \left\{ J \sum_{\alpha=x,y} V\left[\phi(\mathbf{n}+\mathbf{d}_\alpha) - \phi(\mathbf{n}) - a_\alpha(\mathbf{n})\right] - \frac{J}{\gamma^2}\cos\left[\phi(\mathbf{n}+\mathbf{d}_z) - \phi(\mathbf{n})\right] \right\}, \quad (1)$$

The energy scale of the system is the phase stiffness, J, definied as

$$J = \frac{s\Phi_0^2}{\pi(4\pi\lambda_{ab})^2} \equiv \frac{s\varepsilon_0}{\pi}, \quad (2)$$

where s is the layer spacing and we have introduced the scale for the vortex line energy $\varepsilon_0 = (\Phi_0/4\pi\lambda_{ab})^2$ with Φ_0 being the flux quantum and λ_{ab} is the penetration depth for supercurrents flowing in the ab-planes. The anisotropy is given by $\gamma = \lambda_c/\lambda_{ab}$, where λ_c is the penetration depth for supercurrents flowing the direction along the c-axis. The phase $\phi(\mathbf{n})$ is defined on a cubic lattice with the lattice spacing s, the dimensionless vector potential is defined as $\mathbf{a} = (0, 2\pi f n_x/\Phi_0, 0)$, with f being the fraction of the lattice cells filled by vortices, and $\mathbf{d}_\alpha$ are unit vectors. The phase interaction $V(\phi)$ is a 2π-periodic function with the Taylor expansion $V(\phi) - V(0) \approx \phi^2/2$ as $\phi \to 0$. An obvious choice is to use $V(\phi) = -\cos(\phi)$, but this produces large barriers for the motion of the vortices between the lattice cells. A natural improvement is therefore to choose a function which minimizes these barriers, as has been done in Ref. 29.

The Bose model, on the other hand, describes the vortex system entirely in terms of the vortex degrees of freedom. A vortex is treated as an elastic string, interacting with the other lines through a *screened* Coulomb potential represented by the modified Bessel function $\mathrm{K}_0(R/\lambda_{ab})$. The free energy functional is then given by[17]

$$\mathcal{F}[\mathbf{R}_i(z)] = \int_0^{L_z} dz \left\{ \sum_i \frac{\varepsilon_l}{2}\left(\frac{d\mathbf{R}_i}{dz}\right)^2 + \sum_{i\neq j} \varepsilon_0 \mathrm{K}_0\left(\frac{R_{ij}}{\lambda_{ab}}\right) \right\}, \quad (3)$$

with L_z is the thickness of the sample, $\mathbf{R}_i(z)$ is a two-dimensional vector describing the position of the vortex, and ε_l is elasticity of a vortex line. This single-line elasticity is an effective quantity, which we choose to reproduce the relevant tilt modulus of the vortex lattice. Two effects complicate the relation between this parameter and the parameters for superconductors. First, it is well known that the tilt energy of vortex line is nonlocal and the effective line stiffness for deformations with wave vector k_z is proportional to $\ln(\gamma/k_z\xi)$. The k_z relevant for melting are given by $k_z \sim \gamma/a_0$. This leads to the estimate[17]

$$\varepsilon_l \approx \frac{\varepsilon_0}{\gamma^2}\ln\left(\frac{a_0}{2\sqrt{\pi}\xi}\right), \quad (4)$$

where a_0 is the lattice spacing of a triangular lattice, i.e., $\Phi_0/B = \sqrt{3}a_0^2/2$. The second effect becomes important in the temperature range close to the fluctuation region where phase and vortex fluctuations start to suppress the Josephson coupling. This suppression enhances the anisotropy and reduces the effective line tension of vortices ε_l as compared to the estimate (4). In the following we introduce the effective anisotropy parameter ε of the Bose model as $\varepsilon^2 \equiv \varepsilon_l/\varepsilon_0$, bearing in mind that $\varepsilon \sim 1/\gamma$, with the main difference coming from the logarithm in Eq. (4).

Both models have their limitations in describing real superconductors. The Bose model describes a continuous anisotropic system whereas the XY-model takes the layered structure of the superconductor into account. Thus one would not expect the Bose model to be applicable when the typical wavelength in the z-direction becomes comparable to the layer spacing. As will be shown below, the correlation length in the z-direction for the Bose model is $l_z = \alpha a_0/\gamma$, where $\alpha \approx 6$ is a numerical constant.[31] Therefore, we expect the Bose model to be invalid when $a_0 < \gamma s/\alpha$ or for $B \gtrsim \alpha^2 B_{\rm cr}$. Numerical results show that line melting works at least for fields as large as $10B_{\rm cr}$.[29] For larger fields, the Josephson coupling between the layers is strongly suppressed by thermal fluctuations, invalidating the use of a simple line model.

Both models neglect the electromagnetic coupling between the superconducting layers. This coupling is relevant for very anisotropic materials when $\gamma s > a_0, \lambda$.[30] The XY-model further neglects the screening of the vortex interaction due to the magnetic field, which becomes relevant close to T_c when $a_0 \gtrsim \lambda(T)$, and therefore does not describe the reentrant behavior of the melting line which is expected for low fields.

When the melting temperature approaches the fluctuation region thermally activated vortex loops start to influence the thermodynamic properties of superconductor strongly. The Bose model only contains the field induced vortices as degrees of freedom and neglects this effect. The XY-model allows for both Gaussian phase fluctuations and thermally induced vortex loops in addition to the field induced vortices. The extra degrees of freedom give a large contribution to the specific heat of the system: Typical numerical data from the XY-model show a specific heat which grows with temperature and has a broad maximum at T_c. On top of this specific heat there is a small sharp peak due to the first-order melting transition.[21,22] These results agree well with experimental observations.[7,8] The important issue is to understand how the field-induced vortices couple to the other fluctuations. Outside the fluctuation region, the coupling only leads to a weak renormalization of vortex interaction and the effective line energy, as is apparent from a number of observations: To begin with, the shape of the melting line is well reproduced by the Lindemann criterion, which only includes field induced vortices with unrenormalized interactions (see, e.g., Refs. 32 and 2). Furthermore, it has recently been shown that the field and temperature dependence of the discontinuity at the vortex lattice melting transition also can be estimated using a simple line model and the mean-field temperature dependence of $\varepsilon_0(T)$.[33]. On the other hand, recent simulations have shown that if melting takes place in the vicinity of the fluctuation region, then a liquid phase immediatly above the transition is not simply a liquid of interacting elastic lines, as the Bose model assumes, but rather an infinite interconnected cluster, in which individual lines loose their identities.[22] Therefore it is natural to test the adequacy of the line picture and establish boundaries of its applicability by direct comparison of the numerical results of the Bose model with the more microscopic XY-model.

II. UNIVERSAL PROPERTIES. COMPARISON OF NUMERICAL RESULTS

Important conclusions can be drawn already from the relevant scales for energy, length, and magnetic field in the expressions (1) and (3). For the XY-model, the relevant scale for the magnetic field is the crossover field $B_{\rm cr}=\Phi_0/(\gamma s)^2$ and the relevant energy scale is the phase stiffness J.[29] Thus, melting lines for systems with different anisotropy factors should collapse on to one line as long as the field is measured in units of $B_{\rm cr}$ and temperature is measured in units of J, as has indeed been demonstrated experimentally.[34,35] In the region of small fields $B\lesssim B_{\rm cr}$ and beyond the fluctuation region $T\lesssim J$ the lattice is expected to melt in a linelike fashion, i.e., the lines are expected to retain their identity above the transition and the Josephson coupling between the layers is only weakly suppressed. In this regime one can expand the Josephson coupling energy with respect to interlayer phase difference and make a continuous approximation, which leads to an even simpler scaling property: The melting temperature should scale as

$$T_m = A_m J\sqrt{B_{\rm cr}/B}. \tag{5}$$

Simulations show that this scaling indeed works very well for fields $B_{\rm cr}\lesssim B\lesssim 10B_{\rm cr}$ with $A_m\approx 0.33$ even though a noticable suppression of the Josephson coupling is observed near T_m (down to 64% of the bare coupling).[29] The scaling form (5) assumes, in fact, that only vortex degrees of freedom participate in the melting transition and regular phase fluctuations play negligible role at T_m. This approximation breaks down when the melting temperature approaches the fluctuation region, which corresponds to $T_m\gtrsim J$ or $B\lesssim B_{\rm cr}$. Regular phase fluctuations and thermally activated vortex loops suppress both the in-plane phase stiffness and the Josephson interlayer coupling. This leads to a renormalization of vortex interactions and their tilt stiffness. Quantitatively these effects can be characterized by the *helicity moduli* Υ_x and Υ_z in the Meissner state. A natural generalization of the scaling form (5), which takes into account this renormalization, can be obtained by the replacements $J\rightarrow\Upsilon_x$ and $\gamma\rightarrow\gamma_Y$ with $\gamma_Y^2=\Upsilon_z/\Upsilon_x$ being the anisotropy of the helicity moduli. This leads to a generalized scaling relations for melting temperature

$$T_m = \tilde{A}_m\Upsilon_x\sqrt{\Phi_0/B(\gamma_Y s)^2} \tag{6}$$

For most of the phase diagram melting occurs in the region where regular phase fluctuations are weak and one can use formulas for $\Upsilon_{x,z}$ with small fluctuation corrections (see Appendix)

$$\Upsilon_x \approx J\left[1-\frac{V^{(4)}T}{4J}\left(1-\frac{\ln(32\gamma^2)-1}{2\pi\gamma^2}\right)\right], \tag{7}$$

$$\Upsilon_z \approx \frac{J}{\gamma^2}\left[1-\frac{T(\ln(32\gamma^2)-1)}{4\pi J}\right] \tag{8}$$

with $V^{(4)}=-\partial^4V(\theta)/\partial\theta^4$ at $\theta=0$ and $V(\theta)$ is the phase interaction function from Eq. (1). To check the scaling relation (6) we used all availible data on the melting transition for XY models with different anisotropies and filling factors.[20,29,21,23] Fig. 1 shows the dependence of T_m/Υ_x vs $1/\sqrt{f}\gamma_Y$, which according to Eq. (6) should be a straight line. We found that in a suprisingly wide range of fields and anisotropies, including a substantial part of

the fluctuation region, the scaling (6) works very well with $\tilde{A}_m \approx 0.4$. At low fields the generalized scaling relation extends down to $B \approx 0.1 B_{cr}$.

Near the transition point $T_c = \alpha_c J$ the helicity moduli vanish according to the XY-scaling laws

$$\Upsilon_x \approx J \left(\alpha_c - \frac{T}{J} \right)^{2\nu}, \tag{9}$$

$$\Upsilon_z \approx \frac{J}{\gamma^2} \left(\alpha_c - \frac{T}{J} \right)^{2\nu}, \tag{10}$$

with $\nu \approx 1/3$. Note that above representations include the possibility of a temperature dependent J near the mean field transition temperature T_{c0}, $J = J_0(T_{c0} - T)/T_{c0}$. Assuming that the scaling law (6) can be extended to the fluctuation region we conclude that in the vicinity of the transition temperature T_c the melting field should scale as $B_m \propto (T_c - T)^{4/3}$. This scaling indeed describes the behavior of experimental melting line better than the "mean field" scaling $B_m \propto (T_{c0} - T)^2$, which follows from scaling relation (5) (see, Refs. 2,5,6).

The Bose model is able to describe the melting transition for arbitrary ratio a_0/λ. There is no general scaling relation which holds for arbitrary values of this ratio. However, in the region of the high vortex density $a_0 \ll \lambda$, where the interaction can be approximated with a logarithm, the free energy only depends on the dimensionless parameter $\Lambda = T/a_0\sqrt{2\varepsilon_l \varepsilon_0}$.[17] In particular, the melting takes place at $\Lambda_m = 0.0622$[17] giving the following scaling relation for the melting temperature.

$$T_m \approx 0.088 \varepsilon \varepsilon_0 a_0. \tag{11}$$

This scaling form corresponds to the low field scaling of the XY-model given by Eqs. (5) and (6). Rewriting Eq. (5) in terms of a_0 we obtain

$$T_m \approx \frac{0.33}{\pi} \sqrt{\frac{\sqrt{3}}{2} \frac{\varepsilon_0 a_0}{\gamma}} \approx 0.098 \varepsilon_0 a_0/\gamma. \tag{12}$$

Thus, the two models do not only agree on the shape of the melting line, but they also agree quantitatively on the position of the melting transition. The result is consistent with the assumption that, according to Eq. (4), we have $\varepsilon > 1/\gamma$ and we find

$$\varepsilon \approx 1.11/\gamma. \tag{13}$$

For finite range interactions the melting temperature is lowered; simulations with $\lambda = 1.06 a_0$ give $T_m \approx 0.084 \varepsilon \varepsilon_0 a_0$, i.e., the melting temperature is lowered by approximately 5%.[17]

We now compare properties of the line liquid near the transition. Both experiments and simulations show the vortex liquid to be heavily entangled so that the phase coherence in the direction parallel to the applied field is destroyed by melting. The entanglement can be quantified in terms of the transverse wandering of the vortex lines,

$$w(z) = \left\langle [\mathbf{R}(z) - \mathbf{R}(0)]^2 \right\rangle / a_0^2. \tag{14}$$

It is easy to show that we have

$$w(z) = \frac{2T}{\varepsilon_l a_0}\frac{z}{a_0}, \tag{15}$$

for non-interacting lines. Since interactions between the lines are unimportant for short distances, this is also the result for small z in the Bose model. For larger z, the wandering is suppressed due to the interaction between the vortices but we still expect $w(z) \propto z\gamma$. The numerical result is

$$w(z) = 0.27\frac{2T}{\varepsilon_l a_0}\frac{z}{a_0}, \tag{16}$$

implying that the line stiffness is increased by a factor 3.7 due to the interaction with other lines in the liquid. For the XY-model as well as for real superconductors the linear dependence (15) is only approximate due to nonlocality of the line tension. In Fig. 2 we show $w(z)$ for two different values of γ in the XY-model, just above the melting transition. We also show $w(z)$ from the Bose model, where we have chosen ε to give the best fit. This yields,

$$w(z) \approx 1.46z/\varepsilon a_0 \Rightarrow \varepsilon \approx 1.46/\gamma, \tag{17}$$

which again is in reasonable agreement with with Eq. (4). For comparison the plot also shows $w(z)$ in the lattice just below the melting transition. One can see that line wandering agrees very well for the two models also in the lattice state. In particular an, initial growth of $w(z)$ is similar to the liquid state and it starts to saturate at approximately the same length scale along z-direction. For the Bose model no entanglement is observed in crystal state and line wandering saturates at $w(z) \approx 0.15$. For the XY-model $w(z)$ continues to grow due to rare reconnection events, with slope much smaller than in the liquid state. This small slope increases with decreasing anisotropy.

It is convenient to have a single number characterizing the line wandering in the vortex liquid. A suitable quantity is the entanglement length l_e, which is defined as a length at which average line displacement becomes equal to the radius of the Bravais cell,

$$w(l_e) = \frac{\Phi_0}{B\pi a_0^2} \approx 0.276. \tag{18}$$

From Fig. 2 we find an entanglement length $l_e \approx 6.6a_0/\gamma$, implying that the vortex system can become fully entangled already in a thin sample.

We now turn to the thermodynamic properties of the melting transition obtained from simulations. A main characteristic of the melting transition is the latent heat or entropy jump. As is now generally accepted, it is important to take the internal temperature dependence of the free energy functional $\mathcal{F}$ into account when computing the energy of a superconductor using Ginzburg-Landau theory.[28,33,29] As a result, the energy is not just the average of $\mathcal{F}$ but is rather given by

$$E = \langle \mathcal{F} \rangle - T\left\langle \frac{\partial \mathcal{F}}{\partial T} \right\rangle, \tag{19}$$

where $\langle \ldots \rangle$ indicates the Monte Carlo average. In the XY-model, the temperature dependence of $\mathcal{F}$ is only due to the temperature of the energy scale $J = s\varepsilon_0/\pi$. The same is true for the Bose model if we assume a logarithmic interaction between the vortices, i.e., when $\lambda > a_0$. We then obtain the simple result

$$E = \left(1 - \frac{T}{\varepsilon_0}\frac{\partial \varepsilon_0}{\partial T}\right) \langle \mathcal{F} \rangle. \tag{20}$$

This equation, together with numerical data for the discontinuity in $\langle \mathcal{F} \rangle$, produces results which are consistent with experiments.[33]

It should be remembered, however, that the statistical weight of a particular vortex configuration is still given by $\mathcal{F}$. It therefore makes sense to study the *configurational* energy $E_c = \langle \mathcal{F} \rangle$ and the corresponding entropy, $S_c = E_c/T = \langle \mathcal{F} \rangle /T$. It is this configurational entropy which we expect to be proportional to the number of degrees of freedom of the vortex system. In order to be able to compare the results from the two models, we only consider the energy per vortex and length,

$$e_c = \frac{E_c}{NL_z} = \frac{1}{NL_z}\langle \mathcal{F} \rangle. \tag{21}$$

The Bose model gives[17]

$$\Delta e_c \approx 0.013\varepsilon_0 \tag{22}$$

and the XY-model typically produces a jump[29]

$$\Delta e_c \approx \ 0.016\varepsilon_0. \tag{23}$$

It is interesting to note that the jump in the configurational entropy per entanglement length at the transition is

$$\Delta s_c l_e = \Delta e_c l_e / T_m \approx (0.9 - 1.1) k_B. \tag{24}$$

Thus, loosely speaking, the system has gained one degree of freedom per field-induced vortex and entanglement length. The result is again perfectly consistent with linelike melting.

The discontinuity in magnetization can be computed from the entropy jump using the Clausius-Clapeyron relation,

$$\Delta S = -\frac{\Delta B}{4\pi}\frac{dH_m}{dT}, \tag{25}$$

where $H_m(T)$ is the applied external field at melting. If we use the approximation $H_m(T) \approx B(T)$, which again is true for $\lambda > a_0$, we can rewrite this as

$$\frac{\Delta e_c}{\varepsilon_0} = 8\pi\lambda^2\Delta\rho, \tag{26}$$

where $\rho = B/\Phi_0$ is the density of the flux lines. If we further assume the latent heat to be independent of the interaction range, we obtain a simple expression for the magnetization jump,

$$\Delta B \approx 5.2 \times 10^{-4}\, \Phi_0/\lambda^2. \tag{27}$$

This result explains why the magnetization jump vanishes in the limit $\lambda \to \infty$ and is in good agreement with experimental results for YBCO. An advantage with the Bose model is that we can do a simulation with finite λ, allowing the density to fluctuate, and test the consistency of the simulation with Clausius-Clapeyron directly.[17]

It is also interesting to examine what fraction of the latent heat comes from the jump in tilting energy of vortices which corresponds to the Josephson coupling between the layers in the XY model. It was shown that for the Bose model the jump in the average tilt energy Δe_{tilt} is exactly half of the latent heat Δe_c, where $e_{tilt} = \varepsilon_l \langle (d\mathbf{R}/dz)^2 \rangle /2$.[17] This can also be considered as an indication of the linelike regime. To investigate this property for the XY model one has to study the relative jump of the Josephson energy $\Delta e_J/\Delta e_c$ with $e_J = -fJ/(\gamma^2)\langle \cos[\phi(\mathbf{n}+\mathbf{d}_z) - \phi(\mathbf{n})]\rangle$. Simulations shows that for a wide range of fields $\Delta e_J \approx 0.5 \Delta e_c$.[29] Moreover, using the fact that the free energy must be continuous at melting, the following relation can be derived

$$\frac{\Delta e_J}{\Delta e_c} = -\frac{\partial \ln T_m}{\partial \ln \gamma^2}, \tag{28}$$

which shows that this property is a consequence of the scaling relation (5).

To conclude, we have compared and discussed numerical result from two widely used models for the vortex system, the XY-model and the Bose model. We have shown that the models agree even quantitatively over a large part of the phase diagram, where we conclude the melting to be linelike. We have identified a number of universal properties, the entanglement length and configurational entropy per entanglement length, which we believe are generic to the transition in the linelike regime.

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III. APPENDIX: FLUCTUATION CORRECTIONS TO HELICITY MODULI IN XY-MODEL

The helicity moduli Υ_x and Υ_z describe the supercurrent responses to phase rotations. In the Meissner state they are determined by the following thermodynamic averages

$$\Upsilon_x = J\left\langle V''\left(\phi(\mathbf{n}+\mathbf{d}_x) - \phi(\mathbf{n})\right)\right\rangle - \frac{J^2}{TN}\left\langle \left[\sum_{\mathbf{n}} V'\left(\phi(\mathbf{n}+\mathbf{d}_x) - \phi(\mathbf{n})\right)\right]^2 \right\rangle, \tag{29}$$

$$\Upsilon_z = \frac{J}{\gamma^2}\left\langle \cos\left(\phi(\mathbf{n}+\mathbf{d}_z) - \phi(\mathbf{n})\right)\right\rangle - \frac{J^2}{TN\gamma^4}\left\langle \left[\sum_{\mathbf{n}} \sin\left(\phi(\mathbf{n}+\mathbf{d}_z) - \phi(\mathbf{n})\right)\right]^2 \right\rangle, \tag{30}$$

where N is the total number of grid sites, $V' = dV(\phi)/d\phi$, $V'' = d^2V(\phi)/d\phi^2$, and $\langle \ldots \rangle$ notates average with the energy functional (1). At finite temperatures the helicity moduli are suppressed by fluctuations below their zero temperature values J and J/γ^2

$$\Upsilon_x \approx J\left(1 - \frac{V^{(4)}}{2}\left\langle (\phi(\mathbf{n}+\mathbf{d}_x) - \phi(\mathbf{n}))^2 \right\rangle\right) \tag{31}$$

$$\Upsilon_z \approx \frac{J}{\gamma^2}\left(1 - \frac{1}{2}\left\langle (\phi(\mathbf{n}+\mathbf{d}_z) - \phi(\mathbf{n}))^2 \right\rangle\right) \tag{32}$$

where $V^{(4)} = d^4V(\phi)/d\phi^4|_{\phi=0}$. Phase fluctuations at small temperatures are determined by the Gaussian energy functional which is obtained by expansion of Eq.(1)

$$\mathcal{F}[\phi(\mathbf{n})] = \frac{J}{2}\sum_{\mathbf{n}}\left[\sum_{\alpha=x,y} (\phi(\mathbf{n}+\mathbf{d}_a) - \phi(\mathbf{n}))^2 + \frac{1}{\gamma^2}(\phi(\mathbf{n}+d_z) - \phi(\mathbf{n}))^2\right] \tag{33}$$

Using this functional we calculate

$$\left\langle (\phi(\mathbf{n}+d_x) - \phi(\mathbf{n}))^2 \right\rangle = \frac{T}{2J}\left(1 - \frac{1}{2\pi\gamma^2}\left(\ln 32\gamma^2 - 1\right)\right) \tag{34}$$

$$\left\langle (\phi(\mathbf{n}+d_z) - \phi(\mathbf{n}))^2 \right\rangle = \frac{T}{2\pi J}\left(\ln 32\gamma^2 - 1\right) \tag{35}$$

Substituting these expressions into Eqs. (31) and (32) we obtain Eqs. (7) and (8) of the paper.

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FIGURES

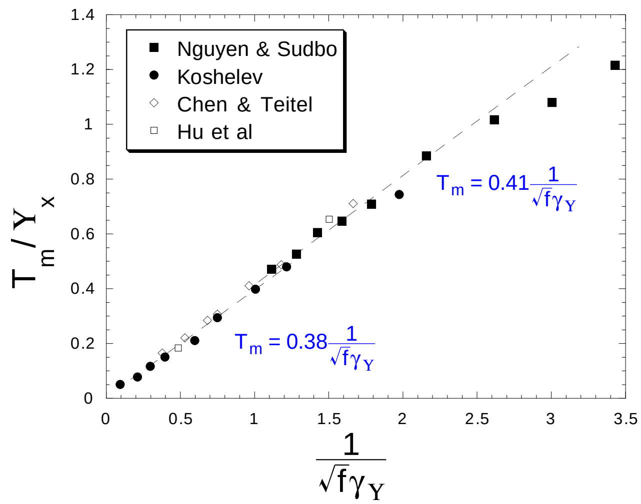

FIG. 1. Scaling plot of melting temperature in the line melting regime for XY models with different anisotropy factors. Data are taken from Refs. 20,29,21,23. Linear fits are made for data from Refs. 29,23.

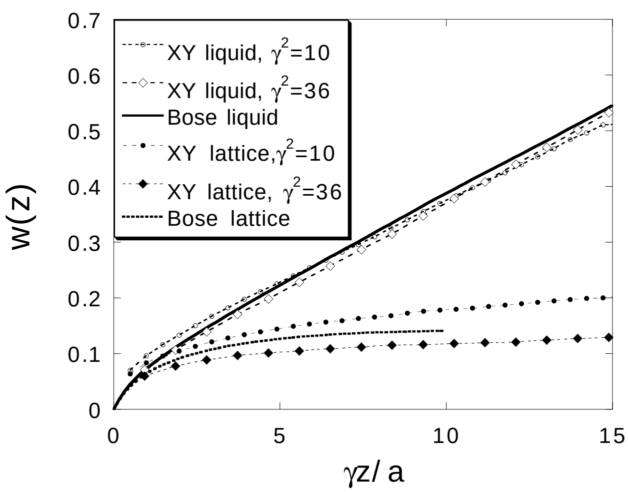

FIG. 2. Comparison of line wandering in the crystal and liquid states near the melting point for the Bose model and the frustrated XY-model. The two models show very similar behavior both with regards to the asymptotic and the initial slope in the liquid. Likewise, the saturation of the line wandering in the crystal is captured correctly by both models.